

Projection methods for the Western Australia, Capital City/Rest of State, Statistical Area 2s (SA2s) and Statistical Area 1s (SA1s) - more details

This approach gives a more detailed breakdown of the four-tiered approach outlined in Appendix I. Apart from the births formulae all equations apply to both sexes, so sex has not been denoted. "State" and "state-level" may refer to either State or Territory.

Step 1 - State/Territory/Australia Projections

This involved projecting the Western Australian population by age and sex at 30 June 2013 out to 30 June 2017.

The cohort component method used can be summarised in the formulae below:

x	-> age
max	-> highest age projected (100+ for state; 85+ for sub-state)
t	-> base year
P	-> population
F	-> fertility rate
f	-> females
B	-> births
Q	-> death probability
OM	-> net overseas migration
IM	-> net interstate (or internal) migration
NM	-> net migration (SA2 projections only)

In Step 1 the following refer to interstate migration;

Step 2 they refer to internal migration;

Step 3 they refer to overseas + inter-SA2 migration.

DEP	-> departures
ARR	-> arrivals
DEPRATE	-> per capita departure rate (donor state <i>or</i> capital city-rest <i>or</i> SA2)
ARRRATE	-> per capita arrival rate (receiving states)

For ages 0 to maximum age - 1:

$$(i) \quad P_{x+1}(t+1) = P_x(t) * [1 - Q_x(t)] + \\ (0.5 * OM_x(t)) * (1 - (0.5 * Q_x(t))) + \\ (0.5 * OM_{x+1}(t)) * (1 - (0.5 * Q_{x+1}(t)))$$

$$\begin{aligned}
(ii) \quad P_{\max}(t+1) &= P_{\max}(t) * [1 - Q_{\max}(t)] + \\
&P_{\max-1}(t) * [1 - Q_{\max-1}(t)] + \\
&OM_{\max}(t) * (1 - (0.5 * Q_{\max}(t))) + \\
&(0.5 * OM_{\max-1}(t)) * (1 - (0.5 * Q_{\max-1}(t)))
\end{aligned}$$

Births were then calculated:

$$(iii) \quad B(t) = 0.5 * [\sum_{x=15}^{49} (F_x(t) * P_{f,x}(t)) + \sum_{x=15}^{49} (F_x(t+1) * P_{f,x}(t+1))]$$

After constraining to projected Australian-level births, these were then used to calculate age 0 in the projected year:

$$(iv) \quad P_0(t+1) = B(t) * (1 - Q_0(t)) + (0.5 * OM_0(t)) * (1 - (0.5 * Q_0(t)))$$

Interstate migration was calculated by applying departure rates to the Western Australian population and arrival rates to the population of the remaining States and Territories (to obtain numbers departing other States to reside in Western Australia). These rates were based on the assumptions published in *Population Projections, Australia, 2012 (base) -2101* (ABS Cat. No. 3222.0).

$$(v) \quad DEP_x(t+1) = P_x(t+1) * DEPRATE_x$$

$$(vi) \quad ARR_x(t+1) = P_x(t+1)^{Non-WA} * ARRRATE_x$$

The resulting total arrivals and departures were then scaled to a predetermined total net interstate migration assumption. Finally, the arrivals and departures by age and sex were scaled to the new arrival and departure totals, then combined to give net age/sex interstate migration.

$$(vii) \quad IM_x(t+1) = ARR_x(t+1) - DEP_x(t+1)$$

Then add the interstate migration:

$$(viii) \quad P_x(t+1) = P_x(t+1) + IM_x(t+1)$$

To achieve coherent interstate migration figures, projections are concurrently run for all States, Territories and Australia. After constraining of State age/sex population sum to the Australian-level (method described in Step 2), year t+1 then became the base for projecting the next year and the cycle was repeated until the final projection year was reached.

Step 2 - Perth / Rest of WA Projections

This employs the cohort component method to project the Perth Greater Capital City Statistical Area and the Rest of Western Australia. The formulae in Step 1 generally apply to these projections, except that the upper age is 85+, fertility rates are by 5yr age of mother and migration arrival levels are used instead of rates.

For ages 0 to maximum age - 1:

$$(ix) \quad P_{x+1}(t+1) = P_x(t) * [1-Q_x(t)] + \\ (0.5 * OM_x(t)) * (1-(0.5 * Q_x(t))) + \\ (0.5 * OM_{x+1}(t)) * (1-(0.5 * Q_{x+1}(t)))$$

$$(x) \quad P_{max}(t+1) = P_{max}(t) * [1-Q_{max}(t)] + \\ P_{max-1}(t) * [1-Q_{max-1}(t)] + \\ OM_{max}(t) * (1-(0.5 * Q_{max}(t))) + \\ (0.5 * OM_{max-1}(t)) * (1-(0.5 * Q_{max-1}(t)))$$

Births were then calculated:

$$(xi) \quad B(t) = 0.5 * [\sum_{x=15-19}^{45-49} (F_x(t) * P_{f,x}(t)) + \sum_{x=15-19}^{45-49} (F_x(t+1) * P_{f,x}(t+1))]$$

After constraining to projected State-level births, these were then used to calculate age 0 in the projected year:

$$(xii) \quad P_0(t+1) = B(t) * (1-Q_b(t)) + (0.5 * OM_0(t)) * (1-(0.5 * Q_0(t)))$$

Capital city-Rest of state internal migration departures were calculated by applying 2006 Census-derived departure rates to the population:

$$(xiii) \quad DEP_x(t+1) = P_x(t+1) * DEPRATE_x$$

Total capital city-Rest of state internal arrivals were then derived using the pre-set net migration assumptions:

$$(xiv) \quad ARR(t+1) = NM(t+1) - \sum_{x=0}^{x=max} DEP_x(t+1)$$

- (xv) The assumed age-specific arrival levels were derived from 2011 Census data. Together with departures from (xiii) these were simultaneously constrained (via IPF - see xvii - xix) to:
- (a) Capital city-Rest of state arrival and departure totals
 - (b) State age-specific net migration

Then the arrivals and departures were applied to the population projected so far:

$$(xvi) \quad P_x(t+1) = P_x(t) + ARR_x(t+1) - DEP_x(t+1)$$

Year t+1 then became the base for projecting the next year and the cycle was repeated until the final projection year was reached. However, before $P_x(t+1)$ became the new base, the projected capital city-rest of state were constrained to sum to the State projection. This involved a final 2-way iterative proportional fitting (IPF) process; the year is t+1:

CC-Rest -> Capital City or Rest of State *region*
 WA -> Western Australia
 a -> first region
 z -> last region
 r -> region number

Scale the regional (capital city-rest of state) totals to the State total:

$$(xvii) \quad P^{CC-Bal} = P^{CC-Bal} * (P^{WA} / \sum_{r=a}^{r=z} P_r^{CC-Bal})$$

For each region scale ages to sum to the new region total:

$$(xviii) \quad P_x^{CC-Bal} = P_x^{CC-Bal} * (P^{CC-Bal} / \sum_{x=0}^{x=max} P_{xR}^{CC-Bal})$$

For each age, scale both regions to sum to the State total:

$$(xix) \quad P_x^{CC-Bal} = P_x^{CC-Bal} * (P_x^{WA} / \sum_{r=a}^{r=z} P_{xR}^{CC-Bal})$$

Stages (xviii) and (xix) were then iterated several times before the resulting matrix was rounded while not changing the marginal constraints.

Step 3 – Statistical Area 2 Projections

This used the cohort component method to project all Western Australian SA2s. The formulae in Step 1 generally apply to the SA2 projections, except that the upper age is 85+, fertility rates are by 5yr age of mother, migration arrival rates were not used and Net Migration (overseas + inter-SA2) was used instead of overseas and inter-SA2 separately.

This slightly simpler approach to migration was warranted as the overseas component is negligible in most SA2s in comparison with inter-SA2 migration. Furthermore as an annual historical time-series only exists at the SA2 level for *net* migration, any overseas/inter-SA2 split can only be approximated using past Census data.

For ages 0 to maximum age - 1:

$$(xx) \quad P_{x+1}(t+1) = P_x(t) * [1 - Q_x(t)]$$

$$(xxi) \quad P_{\max}(t+1) = P_{\max}(t) * [1 - Q_{\max}(t)] + \\ P_{\max-1}(t) * [1 - Q_{\max-1}(t)]$$

Births were then calculated:

$$(xxii) \quad B(t) = 0.5 * \left[\sum_{x=15-19}^{45-49} (F_x(t) * P_{f,x}(t)) + \sum_{x=15-19}^{45-49} (F_x(t+1) * P_{f,x}(t+1)) \right]$$

After constraining to projected capital city/rest of state births, these were then used to calculate age 0 in the projected year:

$$(xxiii) \quad P_0(t+1) = B(t) * (1 - Q_0(t))$$

SA2 migration departures were calculated by applying 2011 Census-derived departure rates to the population:

$$(xxiv) \quad DEP_x(t+1) = P_x(t+1) * DEPRATE_x$$

Total SA2 arrivals were then derived using the pre-set net migration assumptions:

$$(xxv) \quad ARR(t+1) = NM(t+1) - \sum_{x=0}^{x=\max} DEP_x(t+1)$$

(xxvi) The assumed age-specific arrival levels were derived from 2011 Census data. Together with departures from (xxiv) these were simultaneously constrained (via IPF - see xvii - xix) to:

- (a) SA2 arrival and departure totals (from the previous 2 steps)
- (b) capital city/rest of state age-specific net internal migration

Then the arrivals and departures were applied to the population projected so far:

$$(xxvii) \quad P_x(t+1) = P_x(t) + ARR_x(t) - DEP_x(t)$$

After constraining the SA2 age/sex populations to sum to the capital city/rest of state projections using iterative proportional fitting (method described in Step 2), year t+1 then became the base for projecting the next year and the cycle was repeated until the projection horizon was reached.

Step 4 - Statistical Area 1 Projections

This involved splitting the completed SA2 population projections into SA1.

- (xxviii) Each SA1's ERP aged 18 and over was extrapolated linearly to 30 June 2017, based on 30 June 2011 – 30 June 2013 data.
- (xxix) Results were then aligned so they summed to the SA2 projections. Two approaches were used for this:
 - (a) If extrapolated SA1s sum to less than projected SA2s (or both projection & extrapolation falling) then scale all SA1s in the SA2 pro rata.
 - (b) If the extrapolation was growing faster than the projection, scale down only the growth SA1s according to their share of the growing SA1s.

This dual approach improved the results for SA1s in SA2s where there was widely divergent SA1 growth.