
Appendix II – *Text provided by the Australian Bureau of Statistics*

Projection methods for New South Wales, Capital City/Balance of State, Statistical Local Areas (SLAs) and Census Collection Districts (CDs) - more details

The four-tiered approach outlined in Appendix I has been further disaggregated in this accompanying paper. Apart from the births formulae all equations apply to both sexes, so sex has not been denoted. "State" and "state-level" may refer to either State or Territory.

Step 1 - State/Territory/Australia Projections

This involved projecting the New South Wales population by age and sex, 2009- 2014. The cohort component method used can be summarised in the formulae below:

x	-> age
max	-> highest age projected (100+ for state; 85+ for sub-state)
t	-> base year
P	-> population
F	-> fertility rate
f	-> females
B	-> births
Q	-> death probability
OM	-> net overseas migration
IM	-> net interstate (or internal) migration
NM	-> net migration (SLA projections only)

In Step 1 the following refer to interstate migration;
Step 2 they refer to internal migration;
Step 3 they refer to overseas + inter-SLA migration.

DEP	-> departures
ARR	-> arrivals
DEPRATE	-> per capita departure rate (donor state or capital city-balance or SLA)
ARRRATE	-> per capita arrival rate (receiving states)

For ages 0 to maximum age - 1:

$$\begin{aligned} \text{(i)} \quad P_{x+1}(t+1) &= P_x(t) * [1 - Q_x(t)] + \\ &\quad (0.5 * OM_x(t)) * (1 - (0.5 * Q_x(t))) + \\ &\quad (0.5 * OM_{x+1}(t)) * (1 - (0.5 * Q_{x+1}(t))) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad P_{\max}(t+1) &= P_{\max}(t) * [1 - Q_{\max}(t)] + \\ &\quad P_{\max-1}(t) * [1 - Q_{\max-1}(t)] + \\ &\quad OM_{\max}(t) * (1 - (0.5 * Q_{\max}(t))) + \end{aligned}$$

$$(0.5 * OM_{max-1}(t)) * (1 - (0.5 * Q_{max-1}(t)))$$

Births were then calculated:

$$(iii) \quad B(t) = 0.5 * \left[\sum_{x=15}^{49} (F_x(t) * P_{f,x}(t)) + \sum_{x=15}^{49} (F_x(t+1) * P_{f,x}(t+1)) \right]$$

After constraining to projected Australian-level births, these were then used to calculate age 0 in the projected year:

$$(iv) \quad P_0(t+1) = B(t) * (1 - Q_b(t)) + (0.5 * OM_0(t)) * (1 - (0.5 * Q_0(t)))$$

Interstate migration was calculated by applying departure rates to the New South Wales population and arrival rates to the population of the remaining States and Territories (to obtain numbers departing other States to reside in New South Wales). These rates were derived from 1996, 2001 and 2006 Census data and were held constant for the duration of the projection.

$$(v) \quad DEP_x(t+1) = P_x(t+1) * DEPRATE_x$$

$$(vi) \quad ARR_x(t+1) = P_x(t+1)_{Non-NSW} * ARRRATE_x$$

The resulting total arrivals and departures were then scaled to a predetermined total net interstate migration assumption. Finally, the arrivals and departures by age and sex were scaled to the new arrival and departure totals, then combined to give net age/sex interstate migration.

$$(vii) \quad IM_x(t+1) = ARR_x(t+1) - DEP_x(t+1)$$

Then add the interstate migration:

$$(viii) \quad P_x(t+1) = P_x(t+1) + IM_x(t+1)$$

To achieve coherent interstate migration figures, projections are concurrently run for all States, Territories and Australia. After constraining of State age/sex population sum to the Australian-level (method described in Step 2), year t+1 then became the base for projecting the next year and the cycle was repeated until the final projection year was reached.

Step 2 - Sydney / Balance of New South Wales Projections

This employs the cohort component method to project Sydney Statistical Division and the Balance of New South Wales. The formulae in Step 1 generally apply to these projections, except that the upper age is 85+, the base year is 2008, fertility rates are by 5yr age of mother and migration arrival levels are used instead of rates.

For ages 0 to maximum age - 1:

$$(ix) \quad P_{x+1}(t+1) = P_x(t) * [1 - Q_x(t)] + \\ (0.5 * OM_x(t)) * (1 - (0.5 * Q_x(t))) + \\ (0.5 * OM_{x+1}(t)) * (1 - (0.5 * Q_{x+1}(t)))$$

$$\begin{aligned}
 (x) \quad P_{\max}(t+1) = & P_{\max}(t) * [1 - Q_{\max}(t)] + \\
 & P_{\max-1}(t) * [1 - Q_{\max-1}(t)] + \\
 & OM_{\max}(t) * (1 - (0.5 * Q_{\max}(t))) + \\
 & (0.5 * OM_{\max-1}(t)) * (1 - (0.5 * Q_{\max-1}(t)))
 \end{aligned}$$

Births were then calculated:

$$(xi) \quad B(t) = 0.5 * \left[\sum_{x=15-19}^{45-49} (F_x(t) * P_{f,x}(t)) + \sum_{x=15-19}^{45-49} (F_x(t+1) * P_{f,x}(t+1)) \right]$$

After constraining to projected State-level births, these were then used to calculate age 0 in the projected year:

$$(xii) \quad P_0(t+1) = B(t) * (1 - Q_b(t)) + (0.5 * OM_0(t)) * (1 - (0.5 * Q_0(t)))$$

Capital city-balance of State internal migration departures were calculated by applying 2006 Census-derived departure rates to the population:

$$(xiii) \quad DEP_x(t+1) = P_x(t+1) * DEPRATE_x$$

Total capital city-balance of State internal arrivals were then derived using the pre-set net migration assumptions:

$$(xiv) \quad ARR(t+1) = NM(t+1) - \sum_{x=0}^{x=\max} DEP_x(t+1)$$

- (xv) The assumed age-specific arrival levels were derived from 2006 Census data. Together with departures from (xiii) these were simultaneously constrained (via IPF - see xvii - xix) to:
- (a) Capital city-balance of State arrival and departure totals
 - (b) State age-specific net migration

Then the arrivals and departures were applied to the population projected so far:

$$(xvi) \quad P_x(t+1) = P_x(t+1) + ARR_x(t+1) - DEP_x(t+1)$$

Year t+1 then became the base for projecting the next year and the cycle was repeated until the final projection year was reached. However, before $P_x(t+1)$ became the new base, the projected capital city-balance of State were constrained to sum to the State projection. This involved a final 2-way iterative proportional fitting (IPF) process; the year is t+1:

CC-Bal	->	Capital City or Balance of State <i>region</i>
NSW	->	New South Wales
a	->	first region
z	->	last region
r	->	region number

Scale the regional (capital city-balance of State) totals to the State total:

$$(xvii) \quad PCC-Bal = PCC-Bal * (PNSW / \sum_{r=a}^{r=z} PrCC-Bal)$$

For each region scale ages to sum to the new region total:

$$(xviii) \quad PxCC-Bal = PxCC-Bal * (PCC-Bal / \sum_{x=0}^{x=max} PxrCC-Bal)$$

For each age, scale both regions to sum to the State total:

$$(xix) \quad PxCC-Bal = PxCC-Bal * (PxNSW / \sum_{r=a}^{r=z} PxrCC-Bal)$$

Stages (xviii) and (xix) were then iterated several times before the resulting matrix was rounded while not changing the marginal constraints.

Step 3 - Statistical Local Area Projections

This used the cohort component method to project all New South Wales SLAs. The formulae in Step 1 generally apply to the SLA projections, except that the upper age is 85+, the base year 2007, fertility rates are by 5yr age of mother, migration arrival rates were not used and Net Migration (overseas + inter-SLA) was used instead of overseas and inter-SLA separately.

This slightly simpler approach to migration was warranted as the overseas component is negligible in most SLAs in comparison with inter-SLA migration. Furthermore as an annual historical time-series only exists at the SLA level for *net* migration, any overseas/inter-SLA split can only be approximated using past Census data.

For ages 0 to maximum age - 1:

$$(xx) \quad Px+1(t+1) = Px(t) * [1-Qx(t)]$$

$$(xxi) \quad Pmax(t+1) = Pmax(t) * [1-Qmax(t)] + \\ Pmax-1(t) * [1-Qmax-1(t)]$$

Births were then calculated:

$$(xxii) \quad B(t) = 0.5 * (\sum_{x=15-19}^{45-49} [Fx(t) * Pf, x(t)] + \sum_{x=15-19}^{45-49} [Fx(t+1) * Pf, x(t+1)])$$

After constraining to projected capital city/balance of State births, these were then used to calculate age 0 in

the projected year:

$$(xxiii) \quad P_0(t+1) = B(t) * (1 - Q_b(t))$$

SLA migration departures were calculated by applying 2006 Census-derived departure rates to the population:

$$(xxiv) \quad DEP_x(t+1) = P_x(t+1) * DEPRATE_x$$

Total SLA arrivals were then derived using the pre-set net migration assumptions:

$$(xxv) \quad ARR(t+1) = NM(t+1) - \sum_{x=\max} DEP_x(t+1)$$

- (xxvi) The assumed age-specific arrival levels were derived from 2006 Census data. Together with departures from (xxiv) these were simultaneously constrained (via IPF - see xvii - xix) to:
- (a) SLA arrival and departure totals (from the previous 2 steps)
 - (b) capital city/balance of State age-specific net internal migration

Then the arrivals and departures were applied to the population projected so far:

$$(xxvii) \quad P_x(t+1) = P_x(t+1) + ARR_x(t+1) - DEP_x(t+1)$$

After constraining the SLA age/sex populations to sum to the capital city/balance of State projections using iterative proportional fitting (method described in Step 2), year t+1 then became the base for projecting the next year and the cycle was repeated until the projection horizon was reached.

Step 4 - Census Collection District Projections

This involved splitting the completed SLA population projections into Census Collection Districts.

- (xxviii) Each CD's ERP aged 18 and over was extrapolated linearly to July 2012.

- (xxix) Results were then aligned so they summed to the SLA projections. Two approaches were used for this:
- (a) If extrapolated CDs sum to less than projected SLAs (or both projection & extrapolation falling) then scale all CDs in the SLA prorata.
 - (b) If the extrapolation was growing faster than the projection, scale down only the growth CDs according to their share of the growing CDs.

This dual approach improved the results for CDs in SLAs where there was widely divergent CD growth.

Note: CD ERP used AEC data by to incorporate differential growth of CDs when disaggregating any *post-censal* SLA ERP. This affects total CD growth rather than targeting age/sex population change. By constraining to SLA ERP it indirectly 'ages' individual cohorts but resulting CD ERP will tend to reflect the latest Census' age/sex profiles more than would actually be the case in subsequent years. However in this projection where a Census year (2006) is also the latest year of ERP, this is not particularly an issue.